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M(atrix) black holes in five dimensions

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Abstract: We examine five dimensional extreme black holes with three charges in the matrix model. We build configurations of the $5+1$ super Yang–Mills theory which correspond to black holes with transverse momentum charge. We calculate their mass and entropy from the super Yang–Mills theory and find that they match the semi-classical black hole results. We extend our results to nonextreme black holes in the dilute gas approximation.

Keywords: M(atrix) Theories, Black Holes in String Theory.

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Contents

1. Introduction

During the last year and a half our understanding of the microscopic origin of black hole entropy increased enormously. This was achieved by considering $D = 5$ ($D = 4$) black holes with three (four) RR charges and finite entropy at extremality [1, 2]. The main tool that enabled the entropy counting was D brane technology [3]. It was shown that these black holes were built out of different types of D branes (and other charges such as momentum) [4]. In this picture, the microscopic degrees of freedom are the momentum modes which are carried by the massless open strings stretched between the different branes. It was later realized that in the strongly interacting black hole regime the system corresponds to a long string with fractional momentum modes [5]. The above picture can also be described in terms of the SYM theory which describes the world-volume of the D branes. In that description the black hole degrees of freedom correspond to the flat directions of the SYM theory. Surprisingly, it was found that this picture can be generalized to the near-extreme cases when it is not protected by the supersymmetric nonrenormalization theorems [6]. Moreover, Hawking radiation of different types of scalars from black holes was shown to be reproduced in this framework [7, 8].

Since string theory can be derived from M-theory one would like to understand black holes in different dimensions from an M-theoretical point of view. Already (extreme and nonextreme) black holes in different dimensions have been identified as configurations of intersecting M-branes [9]. For example, the five dimensional (extreme) black hole is described by five branes intersecting membranes over a string with momentum flowing on it. The nonextreme black hole is obtained by also allowing antimomentum to flow along the intersecting string. The only candidate for the nonperturbative formulation of M-theory is the M(atrix) theory [10]. In this framework, M-theory in the infinite momentum frame is described by infinitely many longitudinal momentum modes ($\tilde{D}0$ branes) and their interactions due to open strings stretched between them. In order to obtain the eleven dimensional description one takes the limit $N \to \infty, R \to \infty$ with $p = N/R \to \infty$. The lagrangean is given by the dimensional reduction of the $\mathcal{N} = 1$ U(N) SYM theory in $D = 10$ to $0 + 1$ dimensions [11]. Matrix thoery compactified on a torus T^d is described by the dimensional reduction of the above SYM theory to $d+1$ dimensions [12, 10]. However, there is another formulation of the matrix model in the light–cone gauge [13]. In this case, the lagrangean and the degrees of freedom are those in the infinite momentum frame but p, N and R are finite. Finite R is essential for our purposes because we want to describe $D = 5$ black holes by compactifying the matrix model only on T^5 (rather than on T^6 since not much is known about this case). When $R \to \infty$ the five dimensional black hole becomes a black string in six dimensions [14]. Finite N is also relevant since for black hole configurations with longitudinal momentum N is one of the charges and needs to be finite.

Black holes in the framework of matrix model were considered recently [15, 16]. These were black holes with momentum flowing in the light–cone direction. It was shown that one can obtain the correct energy and entropy for these black holes in terms of either an effective string [16] or the world–volume of a NS five brane [15]. This was done in the infinite momentum frame where $N \to \infty$. As a result, these works considered cases with infinite momentum but finite momentum density such that $R \sim N^{1/2}$. However, there are a number of problems with black holes that carry longitudinal momentum. First, one of the charges N diverges which is not the case for a finite black hole. Second, nontrivial effects can take place in the limit $N \to \infty$ compared to the better understood finite N case. Third, in this case one does not know what energy to assign to the rank of the U(N) SYM gauge group (which corresponds to the $\tilde{D}0$ branes) from the SYM point of view. Finally, this is not the only configuration which realizes the $D = 5$ black hole. There are other configurations which are related to the above by rotations in M-theory and by U dualities in IIA string theory.

In this paper, we consider $D = 5$ black holes with three charges from the matrix theoretical point of view. This is done in the finite N formulation of the matrix model which is in the light–cone gauge rather than the original infinite momentum frame. The reason for this is that for finite N the light–cone direction is also compact giving the sixth compact direction required for a matrix model description. For $N \to \infty$ our configurations describe black strings in six dimensions. We briefly mention configurations with momentum in the light–cone direction since this case was already examined. Our main interest is in matrix model configurations with momentum in one of the transverse directions. Matrix theory on T^5 is described by 5+1 U(N) SYM theory and therefore the black hole is a particular configuration of this theory. (For another description of matrix theory on T^5 see [17] where it is argued that this is a tensor theory with $(2,0)$ supersymmetry. In any case, the SYM theory can be taken as the effective low energy theory in the box even if it is not a complete description including ultraviolet effects.) We identify the BPS states of the SYM theory such as electric and magnetic fluxes, instantons, momenta etc. [18] and find configurations which correspond to black holes with transverse momentum charge. These have four BPS charges, the fourth one being the rank of the gauge group which gives the longitudinal momentum of the configuration. We derive a formula for the entropy of these SYM configurations in the box which is not completely U dual (as it does not contain the transverse five brane charge). We find that the rank does not enter the entropy of the black hole for the configurations with transverse momentum. This shows the invariance of black hole entropy under boosts in the light–cone direction as expected. In our opinion these configurations give a clearer picture of five dimensional black holes than the ones with longitudinal momentum. We also calculate the energy of these configurations in the SYM theory which corresponds to the light–cone energy of the black hole. The mass and entropy we find match those of the semiclassical black hole precisely. We extend our calculations to nonextreme black holes in the dilute gas approximation again finding agreement. One case that we cannot apply our results is the configuration with transverse five branes since they do not have a description only in terms of the SYM variables [19, 17] and our entropy formula does not contain their charge.

The outline of the paper is as follows. In section 2, we consider the matrix model compactified on T^5. This is described by a $5+1$ U(N) SYM theory and we find all its BPS states which constitute the black hole. Section 3 is a review of five dimensional black holes with three charges. We consider four different ways of realizing these black holes in matrix theory. In section 4, we find the SYM configurations which correspond to the above realizations. We calculate the mass and entropy of the black holes from the SYM theory and show that they match the semiclassical black hole results. We also consider nonextreme black holes in the dilute gas approximation. In section 5, we discuss our results and their possible implications.

2. M(atrix) theory on T^5

In this section, we review some facts about the matrix model compactified on a five torus. This will be essential for describing black holes in $D=5$ since the five toroidally compactified dimensions together with the light–cone direction (which is compact for finite N) will give us the six compact dimensions of M-theory. In eleven (noncompact) dimensions the matrix model is described by carriers of longitudinal momentum ($\tilde{D}0$ branes) with the lagrangean [10]

$$L = Tr\left(\frac{1}{2R}(D_0X^iD_0X^i) - \frac{1}{4R}[X^i,X^j]^2 + \text{ fermionic terms}\right),$$

where R is the length of the light–cone direction, X^i are $N \times N$ matrices, $D_0 = \partial_0 + iA_0$ and $i = 1, \ldots, 9$.

Matrix theory compactified on T^d is described by a $d+1$ dimensional U(N) SYM theory on a d dimensional finite box [12, 18]. This theory is obtained by dimensionally reducing an $\mathcal{N} = 1$ SYM theory in $9+1$ dimensions to $d+1$ dimensions. For the T^5 compactification, we consider the $5+1$ dimensional SYM theory with the lagrangean

$$L = \int^V d^5\sigma \quad Tr \left(-\frac{1}{4g_6^2} F_{\mu\nu}^2 + (D_\mu X^i)^2 + g_6^2 [X^i, X^j]^2 + \text{ fermionic terms} \right), \quad (2.1)$$

where $V = \Sigma_1 \Sigma_2 \Sigma_3 \Sigma_4 \Sigma_5$ is the volume of the box, g_6 is the gauge coupling constant, $\mu, \nu = 0, \ldots, 5$ and $i, j = 6, \ldots, 9$.

The SYM theory is in a 5-dimensional box with sides (parametrized by $\sigma_1, \ldots, \sigma_5$) of length [20]

$$\Sigma_i = \frac{\ell_{11}^3}{RL_i},$$

where the transverse compact dimensions are of length L_1, L_2, L_3, L_4, L_5 and the light–cone direction is of length R. The dimensionful gauge coupling constant is [19]

$$g_6^2 = \frac{\ell_{11}^9}{R^2 L_1 L_2 L_3 L_4 L_5}.$$

The $D = 5$ IIA string theory has 27 point-like BPS states which are in the fundamental representation of the U duality group E_6. We will identify these BPS states in the SYM theory. These BPS states correspond to the constituents of the different $D = 5$ black holes with three charges. The $5+1$ SYM theory has only 16 BPS states which are the five electric fluxes (the Kaluza–Klein momenta), the ten magnetic fluxes (the wrapped membranes) [18] and a magnetic flux through a plane which is not manifest in the box (the wrapped transverse five brane) [19]. When one considers a compact longitudinal direction (finite N) one can also have five instantonic strings (the longitudinal five branes), five momenta in the SYM box (the longitudinal membranes) and the rank N of the U(N) SYM theory (the longitudinal momenta). Altogether these are 27 BPS states of the $D = 5$ string theory.

The five Abelian electric fluxes which correspond to Kaluza–Klein modes are given by [22]

$$\epsilon_{ijklm} E_i^A \Sigma_j \Sigma_k \Sigma_l \Sigma_m = \frac{2\pi n_i}{N} 1_{N\times N}. \quad (2.2)$$

This is accompanied by a non-abelian electric flux

$$\epsilon_{ijklm} E_i^{NA} \Sigma_j \Sigma_k \Sigma_l \Sigma_m = \frac{2\pi n_i}{N} \omega,$$

where $\omega = \text{diag}(1, 1, 1, \ldots, 1 - N)$ is an SU(N) matrix. Similarly, the ten Abelian magnetic fluxes which describe the wrapped membranes are given by [21, 22]

$$F_{ij}^A \Sigma_i \Sigma_j = \frac{2\pi n_{ij}}{N} 1_{N\times N}, \quad (2.3)$$

with the non-abelian fluxes

$$F_{ij}^{NA}\Sigma_i\Sigma_j = \frac{2\pi n_{ij}}{N}\omega\,.$$

The wrapped transverse five brane is given by the Abelian magnetic flux [19]

$$F_{5\sigma}^{A}\Sigma_5\Sigma = \frac{2\pi n}{N}1_{N\times N}$$

and a non-abelian flux obtained from the above by substituting ω for the unit matrix. Here $\Sigma = g_5^2 = \ell_{11}^6/RL_1L_2L_3L_4$ is the size of a new direction (parametrized by σ) which opens up as $g_5^2 \to \infty$ but is not manifest in the box [23]. The light–cone energy of these states are reproduced only by the Abelian fluxes. The form of ω changes when there are two orthogonal magnetic fluxes or magnetic and elextric fluxes which have a common direction. In these cases, the non-abelian fluxes also contribute to the energy as we will later see in the black hole context. Note that for each BPS state the amount of Abelian and non-abelian fluxes are equal. The five longitudinal membranes are described by photons in the box with momenta [24]

$$p_i = \frac{2\pi m_i}{\Sigma_i}\,.$$

The five longitudinal five branes are given by the instantonic strings (say along σ_5) with energy n/g_5^2 or tension $n/g_6^2 = n/\Sigma\Sigma_5$. The last BPS state is given by the rank N of the U(N) gauge group and corresponds to the $\tilde{D}0$ branes of matrix theory with mass n/R.

3. Five dimensional black holes

In this section, we review the solution for the $D = 5$ black hole with three charges [4]. The classical solution to the low energy equations of motion in type IIB string theory compactified on T^5 is given by the metric $g_{\mu\nu}$, the RR antisymmetric tensor $B_{\mu\nu}$ and the dilaton $g^2 = e^{-2\phi}$. The NS three form, the self-dual five form and the RR scalar are set to zero. Also, the asymptotic value of the dilaton ϕ is taken to be zero. The classical five dimensional (nonextreme) black hole metric is given by

$$ds^2 = -f^{-2/3}\left(1-\frac{r_0^2}{r^2}\right)dt^2 + f^{1/3}\left[\left(1-\frac{r_0^2}{r^2}\right)^{-1}dr^2 + r^2 d\Omega_3^2\right],$$

where

$$f = \left(1+\frac{r_0^2\sinh^2\alpha}{r^2}\right)\left(1+\frac{r_0^2\sinh^2\beta}{r^2}\right)\left(1+\frac{r_0^2\sinh^2\gamma}{r^2}\right).$$

The solution is parametrized by six parameters, $\alpha, \beta, \gamma, r_0$ and the compactified one and four volumes $2\pi R$ and $(2\pi)^4 V$. The total energy of the black hole is

$$E = \frac{RVr_0^2}{2g^2\alpha'^4}(\cosh 2\alpha + \cosh 2\beta + \cosh 2\gamma)\,.$$

The entropy of the black hole is found from the area of the horizon using the Bekenstein–Hawking formula

$$S = \frac{A_H}{4G_5} = \frac{2\pi RV r_0^3}{g^2\alpha'^4}\cosh\alpha\cosh\beta\cosh\gamma\,,$$

where the ten and five dimensional Newton constants are given by $G_{10} = 8\pi^6 g^2\alpha'^4$ and $G_5 = G_{10}/(2\pi)^5 RV$. The black hole carries the three charges

$$\begin{aligned} Q_5 &= \frac{r_0^2}{2g\alpha'}\sinh(2\alpha) \\ Q_1 &= \frac{Vr_0^2}{2g\alpha'^3}\sinh(2\beta) \\ n &= \frac{R^2Vr_0^2}{2g^2\alpha'^4}\sinh(2\gamma)\,. \end{aligned} \tag{3.1}$$

These are the charges of the black hole under the RR three form H_3, its dual H_7 and Kaluza–Klein two form coming from the metric. The extreme black hole limit is obtained by $r_0 \to 0$ and $\alpha, \beta, \gamma \to \infty$ with the charges Q_1, Q_5, n fixed. For the nonextreme case we will consider only the dilute gas approximation which holds for $R \gg \sqrt{\alpha'}$, $V \sim \alpha'^2$ and all charges of the same magnitude, i.e. $Q_5 \sim Q_1 \sim n$ [25]. This is the region of the parameter space in which nonextreme entropy can be reliably calculated. In this regime, when energy is added to the black hole very few anti-D five and one branes are excited compared to the number of antimomenta. Therefore, the dominant contribution to the entropy change comes from the momentum modes.

The properties of the black hole can be written in a suggestive way if we trade the parameters $\alpha, \beta, \gamma, r_0, R, V$ for N_5, N_1, n_L, n_R defined by

$$\begin{aligned} N_5 &= \frac{r_0^2}{4\alpha'}e^{2\alpha} \\ N_1 &= \frac{Vr_0^2}{4g^2\alpha'^3}e^{2\beta} \\ n_L &= \frac{R^2Vr_0^2}{4g^2\alpha'^4}e^{2\gamma} \\ n_R &= \frac{R^2Vr_0^2}{4g^2\alpha'^4}e^{-2\gamma}\,. \end{aligned} \tag{3.2}$$

In terms of the above brane numbers numbers, the charges of the black hole are $Q_1 = N_1$, $Q_5 = N_5$, $n = n_L - n_R$. The black hole mass is

$$M_{BH} = \frac{N_5 RV}{g\alpha'^3} + \frac{N_1 R}{g\alpha'} + \frac{1}{R}(n_L + n_R)\,.$$

The entropy can be written as

$$S = 2\pi\sqrt{N_5 N_1}(\sqrt{n_L} + \sqrt{n_R})\,.$$

The extreme limit corresponds to $n_R = 0$.

The microscopic description of the black hole in terms of D branes is as follows [26]. An extreme black hole with charges Q_5, Q_1 and n is described at weak coupling by Q_1 D one branes inside Q_5 D five branes with n units of momentum along the D string. The D strings are confined inside the world–volume of the D five branes and therefore have oscillations in only the four transverse directions. The system can be described by a configuration with one long string of length Q_5Q_1R which is preferred for entropy reasons [5]. This can be interpreted as fractionation of momentum along the string and leads to the correct black hole entropy. From the point of view of the world–volume theory of the D strings this is a SYM theory in $1+1$ dimensions which is described by a CFT with central charge $c = 6Q_5Q_1$ and total momentum n. The entropy of this system is given by (for $c \gg n$) [1]

$$S = 2\pi\sqrt{\frac{cn}{6}}\,,$$

which reproduces the correct black hole entropy. For extreme black holes this weak coupling counting can be reliably extrapolated to strong coupling which is the black hole regime due to supersymmetry. Amazingly the same can be done for nonextreme black holes in the dilute gas approximation and for low energies [6].

We saw that the $D = 5$ black hole in IIB string theory is given by D one branes inside D five branes with momentum along it. In order to make direct contact with M-theory we should T dualize along one direction and pass to a configuration in IIA string theory. If we T dualize along a direction inside the four volume V we get four branes intersecting membranes on a string along which momentum flows. On the other hand, we can T dualize along the R direction and obtain zero branes inside four branes with wound strings orthogonal to the four brane. There are other IIA configurations which are related to these by U dualities. The above configurations in IIA string theory are described in eleven dimensional M-theory by N_5 five branes intersecting N_2 membranes over a string on which there is N_0 units of momentum [9]. Since there are six compact dimensions of the matrix model (the five torus and the light–cone) there are a number of ways this configuration can be realized. We will consider four possibilities which are related by rotations in M-theory and by various U dualities in five dimensional IIA string theory. The four cases are as follows:

1. In this case the (N_5) five branes and the (N_2) membranes are longitudinal with (N_0 units of) momenta in the light–cone direction. This configuration and its S dual have been considered in refs. [15] and [16] respectively. In the notation of [16] it is given by

$$\begin{pmatrix} 11 & 10 & 9 & 8 & 7 & . \\ 11 & . & . & . & . & 6 \\ p & . & . & . & . & . \end{pmatrix}.$$

Here the first, second and third rows describe the orientation of the five branes, membranes and momenta respectively. 11 denotes the light–cone direction and

10 is the direction related to the IIA string coupling constant. The momentum modes are the $\tilde{D}0$ branes of matrix theory. In the $5+1$ SYM theory this configuration corresponds to a $\mathrm{U}(N_0)$ theory with N_5 instantonic strings and N_2 units of momenta in the σ_6 direction of the box.

2. This configuration is given by N_5 longitudinal five branes, N_2 membranes along the 10 and 6 directions and N_0 units of momenta along the 10 direction. Schematically

$$\begin{pmatrix} 11 & 10 & 9 & 8 & 7 & . \\ . & 10 & . & . & . & 6 \\ . & p & . & . & . & . \end{pmatrix}$$

Now, momentum modes are the zero branes of IIA string theory. In the SYM theory this configuration is described by four charges; the three above and a fourth (N) one which is the number of $\tilde{D}0$ branes or the longitudinal momentum of the whole system. Thus, we have a $\mathrm{U}(N)$ gauge theory with N_5 instantonic strings along the σ_6 direction, N_2 units of magnetic flux in the $\sigma_6\sigma_{10}$ plane and N_0 units of electric flux in the σ_{10} direction.

3. Another possibility is the configuration

$$\begin{pmatrix} 11 & 10 & 9 & 8 & 7 & . \\ . & . & 9 & . & . & 6 \\ . & . & p & . & . & . \end{pmatrix}$$

This is similar to the second case but it is a different configuration of IIA string theory. Case 2 describes four branes, membranes and strings which was obtained by T dualizing the black hole along the V direction. Case 3, on the other hand, corresponds to the case obtained by T dualizing along the R direction. The SYM picture is obtained by $9 \leftrightarrow 10$ from the above description.

4. Finally, one can also have transverse five branes, longitudinal membranes and momenta along the intersecting string which is along a transverse direction

$$\begin{pmatrix} . & 10 & 9 & 8 & 7 & 6 \\ 11 & . & 9 & . & . & . \\ . & . & p & . & . & . \end{pmatrix}$$

This case is problematic due to the fact that the transverse five brane cannot be expressed solely in terms of the SYM variables in the five dimensional box.

Note that the first configuration has only three charges whereas the others have four charges. The fourth charge corresponds to the rank of the gauge group (the number of $\tilde{D}0$ branes or light–cone momentum). In the next section, we will see that the black hole entropy for cases 2 and 3 is independent of the rank of the group. This is a satisfactory result because it shows that entropy is independent of longitudinal momentum as it should be. Of course, for the first case entropy depends on the rank of the group since there are only three charges and one of them is the momentum charge of the black hole. In our opinion, cases 2 and 3 are better descriptions of the black hole than case 1 due to the manifest invariance of the black hole entropy under longitudinal boosts. Also in case 1 one does not know what energy to assign to the rank of the gauge theory in a box. In cases 2 and 3 there are three BPS states in the SYM theory and they correctly reproduce the mass of the black hole. Another problem with case 1 is that antimomentum states are not allowed since we are in the IMF or the light–cone gauge. This would correspond to a negative rank for the gauge group which has no meaning. This is not a problem in cases 2 and 3 since all three charges in the SYM theory can be negative.

4. M(atrix) black holes in five dimensions

In this section, we calculate the entropy and energy of the SYM configurations we considered in the previous section which correpond to the five dimensional black hole. The first case has been considered in refs. [15] and [16] and we will discuss it very briefly. Our main interest will be in cases 2 and 3. In order to calculate the entropy of the SYM configurations with three different BPS charges, we will first consider the $4+1$ SYM theory and then translate our formula for the entropy to the $5+1$ SYM theory. The energy of the SYM configuration calculated from eq. (2.1) corresponds to the light–cone energy of the black hole.

The entropy of the $4+1$ dimensional U(N) SYM theory with instanton number k and momentum p is well known. This system is described by a superconformal sigma model with the target space $S^{Nk}T^4$ and thus has a central charge $c = 6Nk$ (since there are four bosonic and four fermionic oscillations) [27, 1]. The entropy of the system is given by the usual formula

$$S = 2\pi\sqrt{\frac{cp}{6}} = 2\pi\sqrt{Nkp}\,. \tag{4.1}$$

The BPS charges that enter this formula are the rank of the gauge group (four branes), instanton number (zero branes) and momentum in the box. One can generalize this formula to include the other BPS charges including the longitudinal branes (but not the transverse five brane). In ref. [21] it was shown that when there are two orthogonal magnetic fluxes (membranes) one gets a fractional instanton number given by

$$\nu = k - \frac{n_{ij}n_{kl}}{N}\,,$$

where n_{ij} correspond to the units of magnetic flux in the ij direction as in eq. (2.3). The configuration in the SYM theory becomes

$$F^A_{ij}\Sigma_i\Sigma_j = \frac{2\pi n_{ij}}{N}1_{N\times N}\,, \qquad F^A_{kl}\Sigma_k\Sigma_l = \frac{2\pi n_{kl}}{N}1_{N\times N}\,,$$

with the corresponding non-abelian fluxes

$$F_{ij}^{NA}\Sigma_i\Sigma_j = \frac{2\pi n_{ij}}{N}\omega\,, \qquad F_{kl}^{NA}\Sigma_k\Sigma_l = \frac{2\pi n_{kl}}{N}\omega\,, \tag{4.2}$$

where $\omega = \mathrm{diag}(k,\ldots,k,-l,\ldots,-l)$ and $k+l=N$. These configurations are exactly 't Hooft's toron configurations which describe fractional instantons. In this case, both the Abelian and non-abelian fluxes contribute to the energy of the configuration. This system is known to be a superconformal sigma model with target space $S^{N\nu}T^4$.

In addition, it was shown in ref. [28] that when there are magnetic and electric fluxes with a common direction the momentum in the box becomes fractional

$$p = m_i - \frac{n_j n_{ij}}{N}\,.$$

The SYM configuration is now given by the magnetic fluxes

$$F_{ij}^{A}\Sigma_i\Sigma_j = \frac{2\pi n_{ij}}{N}1_{N\times N}\,, \qquad F_{ij}^{NA}\Sigma_i\Sigma_j = \frac{2\pi n_{ij}}{N}\omega$$

and the electric fluxes

$$\epsilon_{ijklm}E_j^A\Sigma_i\Sigma_k\Sigma_l\Sigma_m = \frac{2\pi n_i}{N}1_{N\times N}$$

and

$$\epsilon_{ijklm}E_j^{NA}\Sigma_i\Sigma_k\Sigma_l\Sigma_m = \frac{2\pi n_i}{N}\omega\,,$$

with the same ω as above. Once again the total energy is a sum over the Abelian and non-abelian contributions.

When both instanton number and box momentum are fractional the system is described by a CFT with the target space $S^{N\nu}T^4$ and total momentum p. The obvious generalization of the entropy formula in eq. (4.1) is

$$S = 2\pi\sqrt{N|(k - n_{ij}n_{kl}/N)||(m_i - n_j n_{ij}/N)|}\,. \tag{4.3}$$

The BPS charges that appear in this formula are the rank N, magnetic fluxes n_{ij}, electric fluxes n_i, box momentum m_i and instanton number k. Considering the $4+1$ SYM theory as the world–volume theory of a four brane these correspond to the number of four branes and membranes, strings, longitudinal membranes and zero branes inside the four branes respectively. Eq. (4.3) for the entropy can be easily generalized to the $5+1$ SYM case giving

$$S = 2\pi\sqrt{N|(k_i - \epsilon_{ijklm}n_{jk}n_{lm}/N)||(m_i - n_j n_{ij}/N)|}\,. \tag{4.4}$$

Considering the $5+1$ dimensional SYM as the world-volume theory of a D five brane the BPS charges N, n_{ij}, n_i, m_i, k_i correspond to five branes and three branes, strings, longitudinal three branes and instantonic D strings inside five branes respectively.

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From the point of view of the original brane configuration in M-theory these are $\tilde{D}0$ branes, membranes, transverse momentum, longitudinal membranes and longitudinal five branes respectively. There is a special direction in the SYM theory which is given by the direction of the instantonic string. The two three branes inside the five brane and the three branes and strings also intersect over the same direction i. This is the origin of the effective string picture of the black hole in matrix theory. We see that using the SYM theory in the box, we obtained the fractionation of momentum and instanton number which is crucial for understanding black hole entropy. Since this is a pure SYM result it can be seen as the matrix theoretical origin of fractionation considering the connection between the compactified matrix theory and SYM theories.

Note that all but one of the 27 BPS charges of the SYM theory we found in section 2 appear in the above formula. The only exception is the transverse five brane charge which does not have a description only in terms of the SYM variables in the box [19, 17]. As a result, the entropy formula is not completely U dual, i.e. E_6 symmetric. This also the reason why we cannot obtain the entropy of the fourth configuration of the last section which includes transverse five branes. In ref. [15] five dimensional black holes with NS charges were examined and a very similar formula was obtained for the entropy. This case corresponds to the description of matrix theory on T^5 by the $(2,0)$ tensor theory. In that case, the entropy of the configuration is due to the noncritical strings which live on T^5 a theory which describes the world–volume theory of NS five branes. These two descriptions are expected to give the same entropy since they are S duals of each other.

We can now calculate the energy and entropy for the different descriptions of the $D = 5$ matrix black hole in terms of the $5+1$ SYM theory. Consider the first case of the previous section. We have longitudinal five branes, longitudinal membranes and momenta along the light–cone direction. In the $5+1$ SYM theory these are described by instantonic strings (N_5), box momenta (N_2) and the rank of the gauge group (N_0). This case was studied in ref. [15] and will not be reviewed here. The black hole entropy is found from eq. (4.4) (using $N = N_0, k = N_5, m_i = N_2$)

$$S_{BH} = 2\pi\sqrt{N_0 N_5 N_2}\,.$$

This matches the semiclassical result given by eq. (3).

The second case is more interesting for our purposes. This configuration is given by N_5 longitudinal five branes intersecting N_2 transverse membranes and N_0 units of momentum along the 10 (the IIA string coupling constant) direction. In the SYM theory these become instantonic strings, magnetic flux and electric flux respectively. The light–cone momentum (number of $\tilde{D}0$ branes) is N and gives the rank of the gauge group U(N). Now, we have three BPS states in the SYM theory and we can find their energies. The N_5 instantons have energy

$$H_{inst} = \frac{1}{g_6^2} Tr \int^V d^5\sigma\ F_{\mu\nu}\tilde{F}_{\mu\nu} = \frac{N_5}{g_5^2}\,.$$

The energy of the electric and magnetic fluxes is given by the sum of the Abelian and non-abelian parts, $H^A + H^{NA}$ where

$$\begin{aligned} H^A &= \frac{1}{2g_6^2} Tr \int^V d^5\sigma\ (F^A_{6,10})^2 + \frac{g_6^2}{2} Tr \int^V d^5\sigma\ (E^A_{10})^2 \\ &= \frac{2\pi^2}{N g_6^2} N_2^2 \frac{\Sigma_7\Sigma_8\Sigma_9}{\Sigma_6\Sigma_{10}} + \frac{2\pi^2 g_6^2}{N} N_0^2 \frac{\Sigma_{10}}{\Sigma_6\Sigma_7\Sigma_8\Sigma_9} \end{aligned} \tag{4.5}$$

and

$$\begin{aligned} H^{NA} &= \frac{1}{g_6^2} Tr \int^V d^5\sigma\ F^{NA}_{0,10} F^{NA}_{6,10} \\ &= \frac{4\pi^2 N_2 N_0}{N} \frac{1}{\Sigma_6}. \end{aligned} \tag{4.6}$$

Here we used the expressions for the electric and magnetic fluxes given by eqs. (5-8) with $n_{6,10} = N_2$, $n_{10} = N_0$. The total energy of the configuration is

$$H = H_{inst} + H^A + H^{NA} \tag{4.7}$$

$$= \frac{N_5}{g_5^2} + \left(\frac{2\pi^2}{N g_6^2}\right) \frac{\Sigma_{10}}{\Sigma_6\Sigma_7\Sigma_8\Sigma_9} \left(\frac{\Sigma_7\Sigma_8\Sigma_9}{\Sigma_{10}} + g_6^2\right)^2. \tag{4.8}$$

This should be equal to the light–cone energy of the black hole. Using the definitions of Σ_i, g_6^2 and the relations $\ell_{str} = \ell_{11}^3/L_{10}$ and $g_{str}^2 = L_{10}^3/\ell_{11}^3$ it is easy to show that the total SYM energy of the configuration is precisely the light–cone energy of the black hole. Note that the mass of the five branes is not squared since they are longitudinal whereas the membrane and zero brane masses are squared since they are transverse.

The entropy of the SYM system is given by eq. (4.4)

$$S = 2\pi\sqrt{N N_5 (N_0 N_2/N)}\,,$$

which is precisely the entropy of the black hole. Note that the entropy does not depend on the light–cone momentum or N as it should be. The third case can be obtained from the above by the interchange $9 \leftrightarrow 10$ and gives the same results.

We can now consider nonextreme black holes in the dilute gas approximation. In order to do so we need to add a small amount of antimomentum (right-handed in the notation of section 3) to the black hole configuration. In terms of the SYM configuration this corresponds to adding a small amount of electric flux in the negative 10 direction such that $\bar{E}_{10} \ll E_{10}$ for both the Abelian and the non-abelian fields.

The total electric field is now $E_{tot} = E_{10} - \bar{E}_{10}$. As a result of the negative electric flux $\bar{E}_{10}$ there is both left and right-handed momentum in the SYM configuration

$$p_L = \frac{n_i n_{ij}}{N}, \qquad p_R = \frac{\bar{n}_i n_{ij}}{N},$$

where $n_i, \bar{n}_i$ correspond to $E_{10}, \bar{E}_{10}$ in the notation of eq. (2.2) respectively. The system is now described by a CFT with the target space $S^{N\nu}T^4$ with fractional p_L and p_R. The entropy is

$$S = 2\pi \left(\sqrt{\frac{c_L p_L}{6}} + \sqrt{\frac{c_R p_R}{6}} \right),$$

with $c_L = c_R = 6$.

5. Discussion and conclusions

In this paper, we examined configurations of the matrix model which correspond to five dimensional black holes. This was done in the light–cone gauge formulation of the model with finite longitudinal momentum and radius of the eleventh dimension. We considered three different configurations that correspond to $D = 5$ black holes: one with longitudinal and two with transverse momentum. We obtained a formula for the entropy of BPS configurations (with three charges) of the $5+1$ dimensional SYM theory which describes the matrix model compactified on T^5. The formula is not completely U dual since it does not contain the transverse five brane charge. Fractionation of momentum and/or instanton number which plays a crucial role in understanding black hole entropy arises automatically from the SYM picture. This can be seen as the origin of fractionation in the (compactified) matrix model. We also calculated the mass of these configurations and found that the energy and entropy of these match those of the black holes precisely. We found that the entropy for the transverse momentum case does not depend on the longitudinal momentum or N. When one takes the limit $N \to \infty$ (since entropy etc. do not depend on N) the black hole becomes a black string in six dimensions. We generalized our results for the nonextreme black holes in the dilute gas approximation. Once again the mass and entropy of the SYM configurations match the black hole results.

The black hole configurations we studied seem to manifest part of the $D = 11$ Lorentz invariance of the matrix model [30]. First, as we saw longitudinal boost invariance was manifest for the black hole configurations with transverse momentum. The difficult part of the eleven dimensional Lorentz invariance is the rotational symmetry between the longitudinal and transverse dimensions. This requires that a given configuration boosted along the light–cone direction is equivalent to the same configuration rotated to a transverse direction (e.g. $9 \leftrightarrow 11$) and boosted along it. This seems to be realized by our black hole configurations as can be seen by comparing case 1 with cases 2 and 3. The mass, charge and entropy of these configurations are the same and therefore this is a manifestation of eleven dimensional Lorentz invariance.

As noted above, the entropy formula we derived for the SYM configurations which correspond to the black holes is not U dual or E_6 symmetric. In particular the transverse five brane charge is missing from the formula. The present description of the five brane requires the use of a new dimension which is not geometrically manifest. Thus, the five brane cannot be described by the SYM and the box variables alone. In order to obtain the entropy of case 4, what seems to be required is either a U dual formula together with a SYM description of the five brane or a generalization of our entropy formula to include the nonmanifest direction. Note that the other description of the transverse five brane in the tensor theory also involves the nonmanifest direction σ [17].

Extending our results to four dimensional black holes does not seem staightforward [31]. First, $D=4$ black holes require six branes (with finite mass) which do not have a description in the matrix model. (Six branes which are built out of three orthogonal stacks of membranes are known but these do not have finite energy [32].) Second, four dimensional black holes require compactifying matrix model on a six torus T^6 which is problematic. In ref. [18] this issue was investigated and it was found that there is no weakly coupled desciption for the T^6 compactifications of the matrix model. This may be related to the lack of a superconformal fixed point in $6+1$ dimensional field theory.

We considered the nonextreme black holes only in the dilute gas approximation (just as in the D brane picture). Can we go beyond this approximation and consider Reissner–Nordstrom black holes for which the deviation from extremality is for the three charges simultaneously? In previous work on matrix black holes and case 1 above this is not possible since one of the charges is the longitudinal momentum. The nonextreme case requires negative momentum which corresponds to negative longitudinal momentum (anti $\tilde{D}0$ branes) which is clearly not possible. In the light–cone description there are no negative longitudinal momentum modes and in the infinite momentum frame these modes decouple from the system. On the other hand, for configurations with momentum in the transverse direction all three charges can be negative and there is no a priori reason why one cannot realize the nonextreme Reissner–Nordstrom case. This hopefully may be a way to go beyond the dilute gas approximation in a reliable manner.

Taking the highly nonextreme (Schwarzschild) limit we may ask whether the matrix model description of black holes enhances our understanding of black hole entropy. In this limit, the D brane picture is not helpful because Schwarzschild black holes are described by highly excited fundamental strings [33, 34, 35]. For large number of antibranes, the branes and antibranes annihilate into strings with no charge. In M-theory fundamental strings are wrapped membranes and therefore states of the SYM theory. The excited (oscillating) string is not a BPS state however.

It is interesting that a highly excited string becomes a black hole when the string coupling is weak ($g \ll 1$ but $gN^p > 1$ for $p < 1$) which is a well understood limit of the compactified matrix model [36]. The string oscillations are described by the nonzero kinetic terms for the scalars in the SYM theory. It seems that the highly excited string configuration can be examined in the matrix model context. On the other hand, the relation between mass and entropy of Schwarschild black holes is known in any dimension. Therefore it would be interesting to consider this limit in matrix model.

A related question is the transition between the weakly coupled collection of SYM BPS states and the strongly coupled configurations which describe the black hole. In the string and D brane context this question was answered by the correspondence principle which states that the transition to the black hole state occurs when the curvature in the string metric is around the string scale [37]. Is there such a principle in the matrix model in terms of the SYM variables? This would require the analogs of concepts such as the string length, black hole radius and curvature etc. in the SYM picture. Unfortunately, the connection between the space-time description of the black hole and the SYM picture is not very clear. For example, among other things, the existence and position of the horizon is not well understood [38].

One possible interpretation of the entropy formula we derived is as follows. The black hole can be described as an effective (long) fundamental string with a given momentum and winding charge and a rescaled tension (due to the presence of a background five brane) [39]. The momentum and winding are fractionized and given eqs. (4.1) and (4.2). The factor N arises due to the rescaling of the string tension i.e. $\alpha'_{eff} \to N\alpha'$. The effective fundamental string is dual to the real D strings (inside the five branes) which are described by the instantons in the SYM theory.

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